

# Eigenvalue Dating Service

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Thanks to Michael Overton for `neardefmat` and

“Characterization and construction of the nearest defective matrix via coalescence of pseudospectral components”

Rafikul Alam

Shreemayee Bora

Ralph Byers

Michael L. Overton

Linear Algebra and its Applications vol. 435, pp. 494-513, 2011.

<https://doi.org/10.1016/j.laa.2010.09.022>

# An Old $3 \times 3$

```
>> % Last Week's Blog
```

```
>> A
```

```
A =
```

```
    -64     82     21  
    144   -178   -46  
   -771    962    248
```

<https://blogs.mathworks.com/cleve/2019/05/22/an-eigenvalue-sensitivity-example/>

```
>> ei g(A)
```

```
3. 0000000000003868  
0. 9999999999998212  
1. 9999999999997978
```

```
>> condei g(A)
```

```
833. 1092  
450. 7228  
383. 7564
```

```
>> [X, ~] = eig(A)
```

```
X =
```

```
    0.0891    0.0735   -0.1089
   -0.1782   -0.1923    0.1634
    0.9800    0.9786   -0.9805
```

```
>> Y = inv(X)
```

```
Y =
```

```
  -521.9612   628.5984   162.7621
   265.1820  -353.5760  -88.3940
  -257.0058   275.3634   73.4302
```

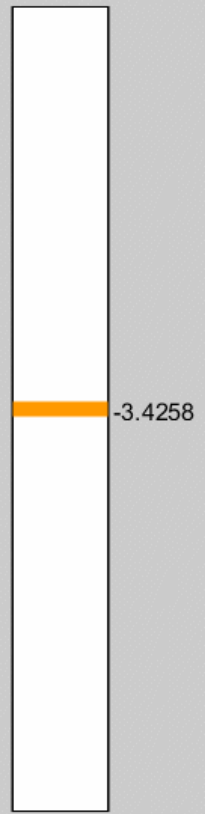
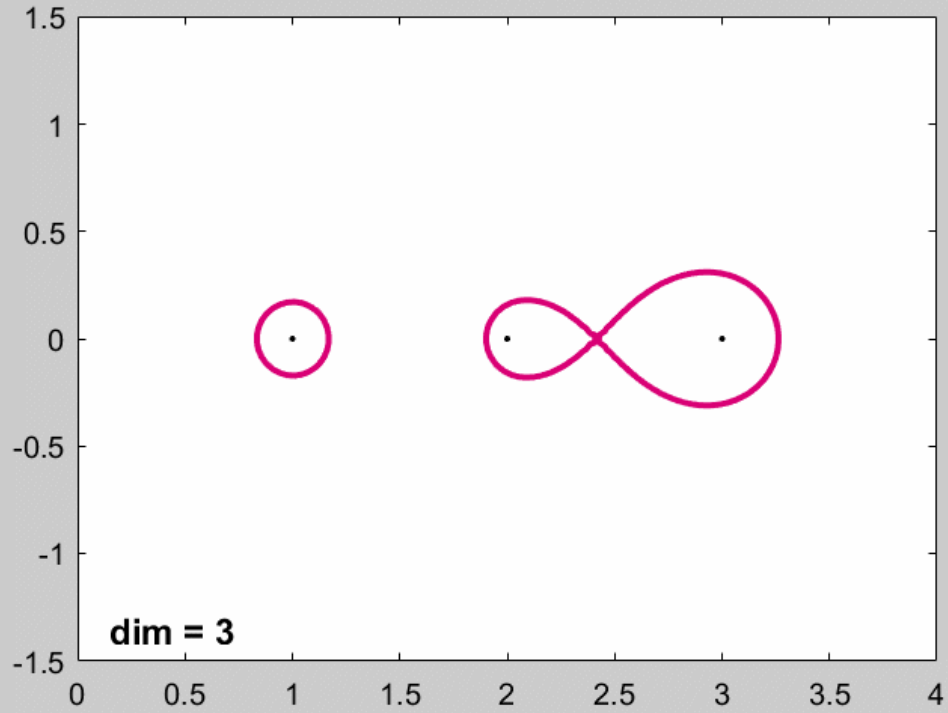
```
>> (vecnorm(Y') .* vecnorm(X))'
```

```
833.1092
```

```
450.7228
```

```
383.7564
```

# EigTool: left button to zoom in, right button to zoom out



Field of Values

Mesh:
   
 Grid Size:

Figure Axes:

Scale Equal

Y max:

X min:  X max:

Y min:

Direct/iterative:

Direct

ARPACK/eigs

No. eigs (k):

Large Mod.

Contour Levels:

log10(largest):

log10(smallest):

Step size:



### EigTool: left button to zoom in, right button to zoom out

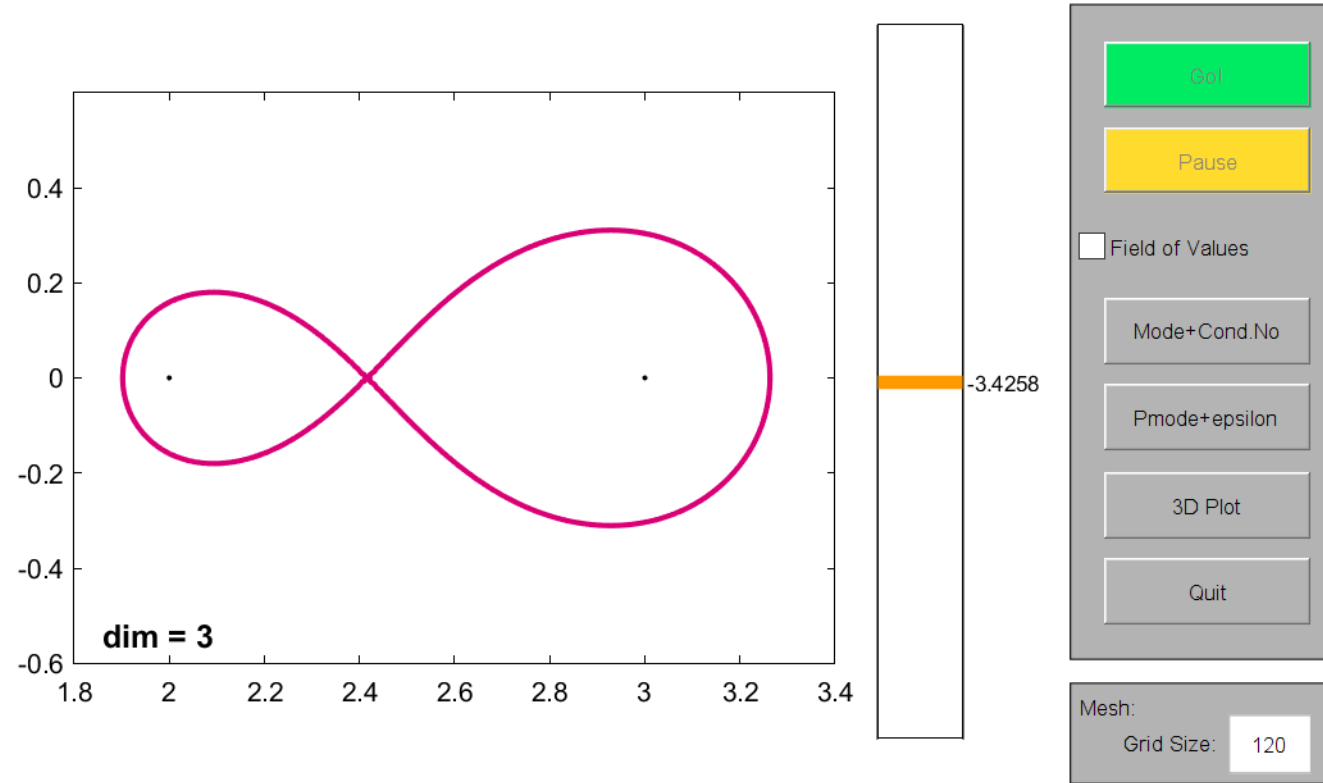


Figure Axes:

Scale Equal

Y max: 0.6

X min: 1.8 X max: 3.4

Y min: -0.6

Direct/Iterative:

Direct

ARPACK/eigs

No. eigs (k): 1

Large Mod.

Contour Levels:

Smart levels

log10(largest): -3.4258

log10(smallest): -3.4258

Step size: 0

### EigTool: left button to zoom in, right button to zoom out

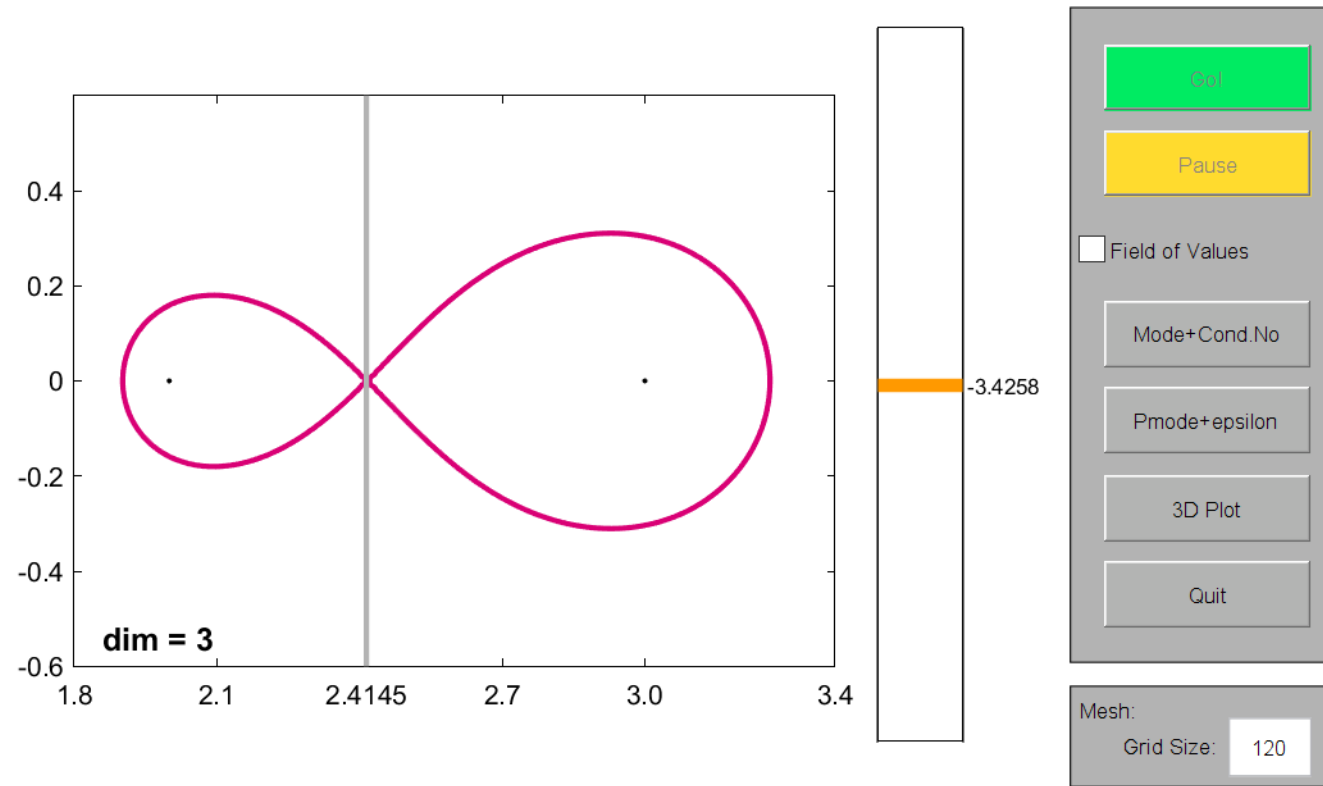


Figure Axes:

Scale Equal

Y max: 0.6

X min: 1.8      X max: 3.4

Y min: -0.6

Direct/Iterative:

Direct

ARPACK/eigs

No. eigs (k): 1

Large Mod.

Contour Levels:

Smart levels

log10(largest): -3.4258

log10(smallest): -3.4258

Step size: 0

```
>> [B, z] = neardefmat(A);
```

```
neardefmat: lower bound on distance is 0.000299834 and upper bound is 0.00618718
```

```
neardefmat: finished, lowest saddle found is 2.41449 + 0i
```

```
with residual
```

```
dist = 0.000375171, mu = 0, |u'*v| = 3.67335e-06
```

```
singular vector norms are 1.8334e-13, 5.17298e-14
```

```
this saddle point was found in search # 3
```

```
distance to nearest defective matrix B found is 0.000375171
```

```
neardefmat: two closest computed eigenvalues of B differ by 0.00269716
```

```
and condition numbers of these eigenvalues are 272230, 272669
```

```
>> z
```

```
z =
```

```
2.414492300748820
```

```
>> [U, S, V] = svd(A - z*I);
```

```
>> sigma = S(3, 3)
```

```
sigma =
```

```
3.751705175569800e-04
```

```
>> u = U(:, 3)
```

```
>> v = V(:, 3)
```

```
u =
```

```
-0.6373
```

```
0.7457
```

```
0.1942
```

```
v =
```

```
0.0941
```

```
-0.1748
```

```
0.9801
```

```
>> B = A - sigma*u*v'
```

```
B =
```

```

-63.9999774916363      81.9999582124016      21.0002343527353
-143.9999736641696    -177.9999511065701    -46.0002742035793
-771.0000068576393    962.0000127314574     247.9999285995845
```

```
>> eig(B)
```

```

2.417189453582737
2.414492295643361
1.168318252152021
```

```
>> norm(B-A)
```

```
ans =
```

```
3.7517e-04
```

Frank(9)



```
>> F = gallery('frank', 9)
```

```
F =
```

9	8	7	6	5	4	3	2	1
8	8	7	6	5	4	3	2	1
0	7	7	6	5	4	3	2	1
0	0	6	6	5	4	3	2	1
0	0	0	5	5	4	3	2	1
0	0	0	0	4	4	3	2	1
0	0	0	0	0	3	3	2	1
0	0	0	0	0	0	2	2	1
0	0	0	0	0	0	0	1	1

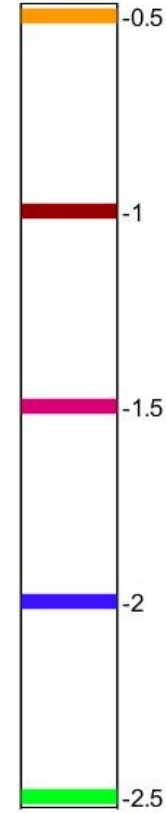
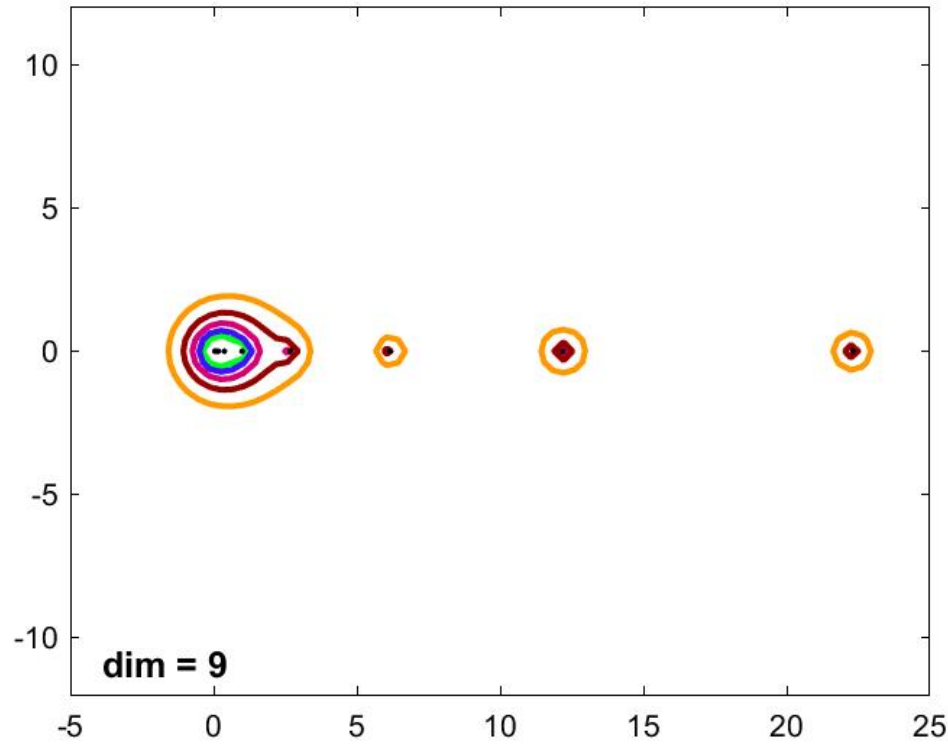
>> eig(F)

0.044802721845067  
 0.082015890201792  
 0.162582729486328  
 0.374121140912600  
 0.9999999999999966  
 2.672931012564413  
 6.150714797051293  
 12.192759202150013  
 22.320072505788492

>> condei g(F)

1.0e+04\*  
 1.476168290732964  
 2.513449461897655  
 1.190700091894456  
 0.159963951495723  
 0.006861536714457  
 0.000362104594905  
 0.000152684676081  
 0.000239032361400  
 0.000199158449368

Press the Go! button to compute on the new axes...



Field of Values  
  
  
  
  
 Mesh:  
 Grid Size:

Figure Axes:

Scale Equal

Y max:

X min:  X max:

Y min:

Direct/iterative:

Direct

ARPACK/eigs

No. eigs (k):

Large Mod.

Contour Levels:

log10(largest):

log10(smallest):

Step size:

Press the Go! button to compute on the new axes...

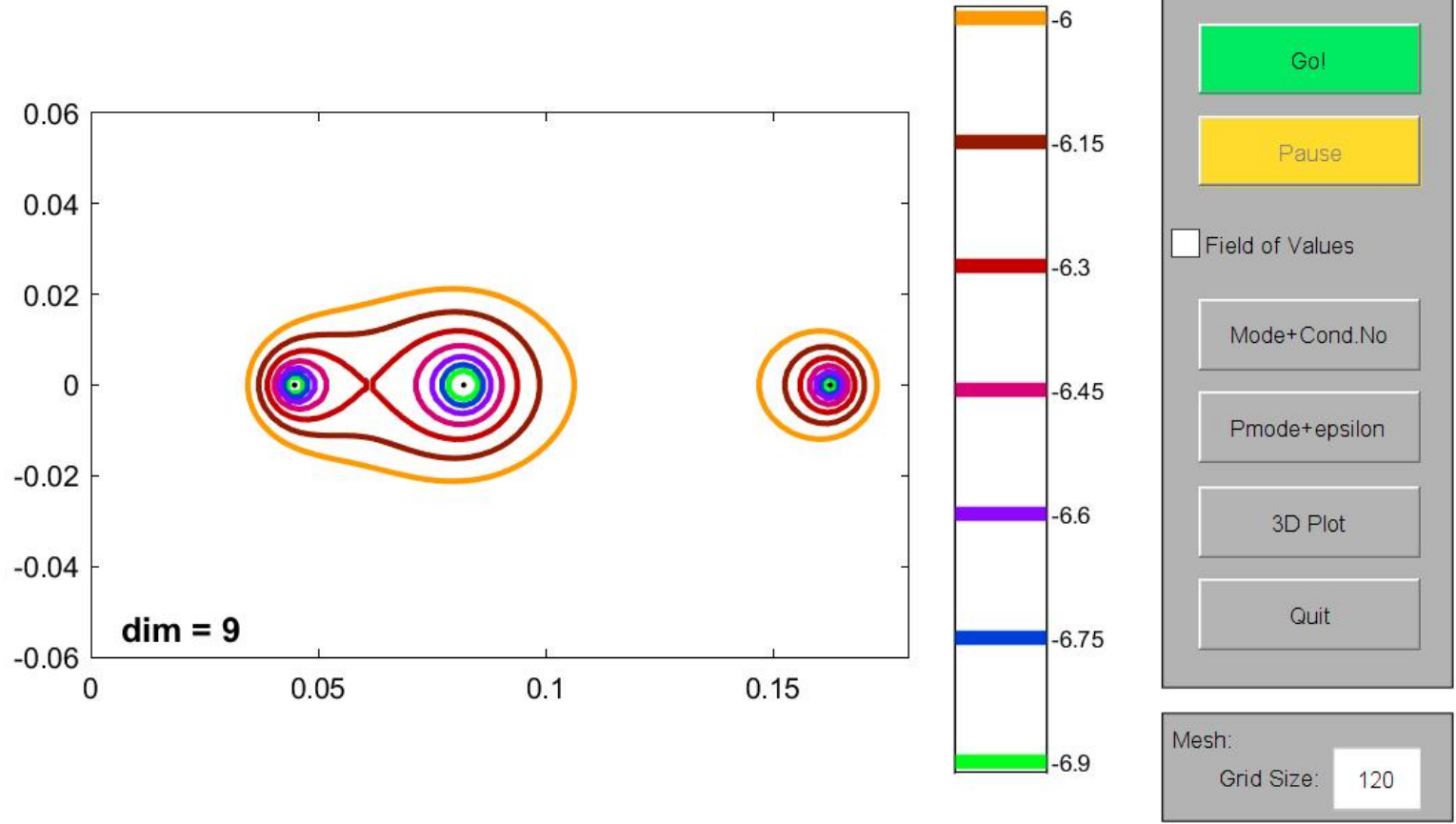


Figure Axes:

Scale Equal

Y max: 0.06

X min: 0      X max: 0.18

Y min: -0.06

Direct/Iterative:

Direct

ARPACK/eigs

No. eigs (k): 6

Large Mod.

Contour Levels:

log10(largest): -6

log10(smallest): -7

Step size: 0.15

```
>> B = neardefmat(F);
```

```
neardefmat: lower bound on distance is 3.4791e-07 and upper bound is 8.76938e-05
```

```
neardefmat: finished, lowest saddle found is 0.06104 + 0 i with
```

```
  dist = 5.01214e-07, mu = 0, |u' * v| = 6.09133e-10
```

```
  singular vector residual norms are 3.6435e-15, 6.12511e-16
```

```
  this saddle point was found in search # 1
```

```
distance to nearest defective matrix B found is 5.01214e-07
```

```
neardefmat: two closest computed eigenvalues of B differ by 1.7808e-06
```

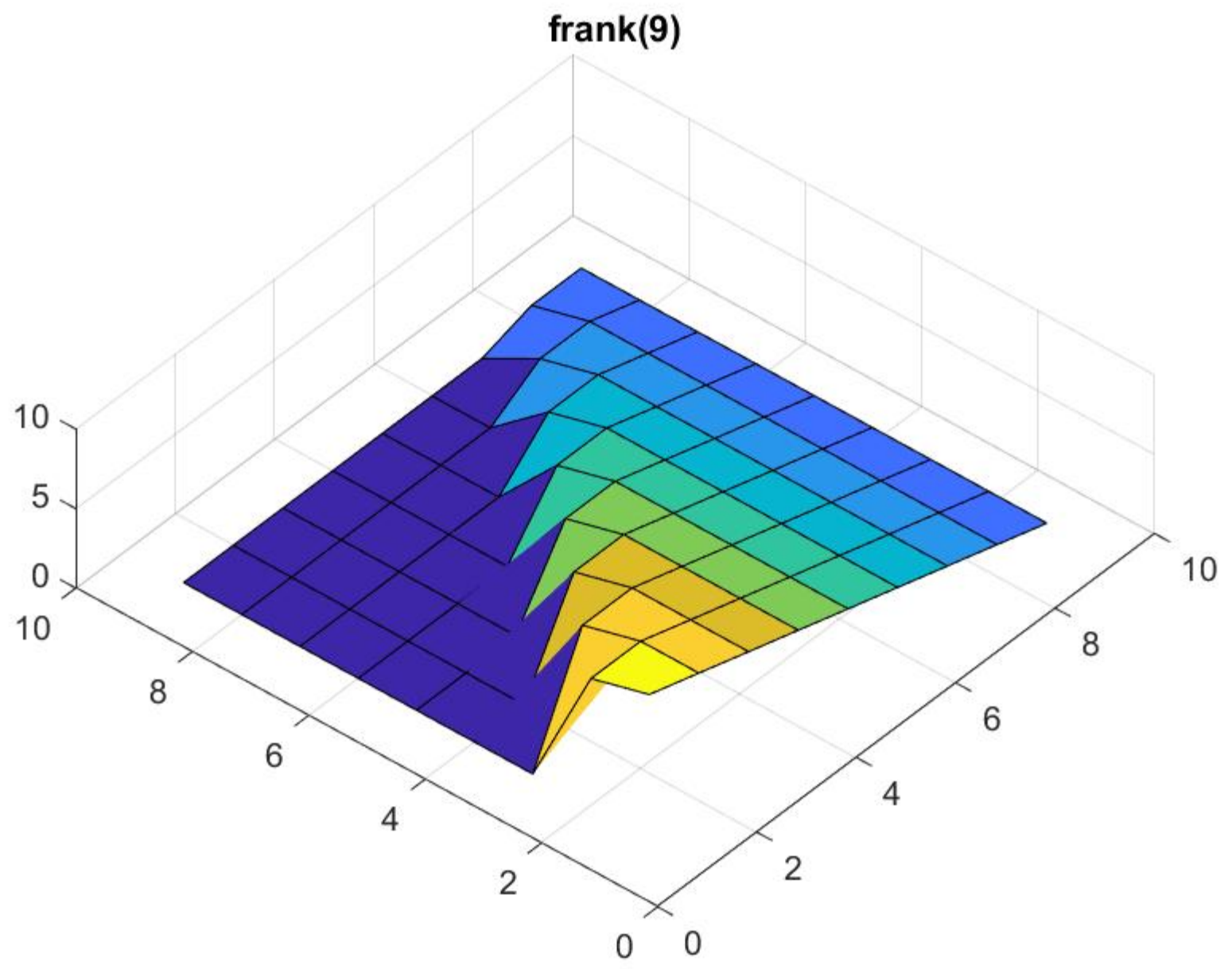
```
  and condition numbers of these eigenvalues are 3.62946e+08, 3.62937e+08
```

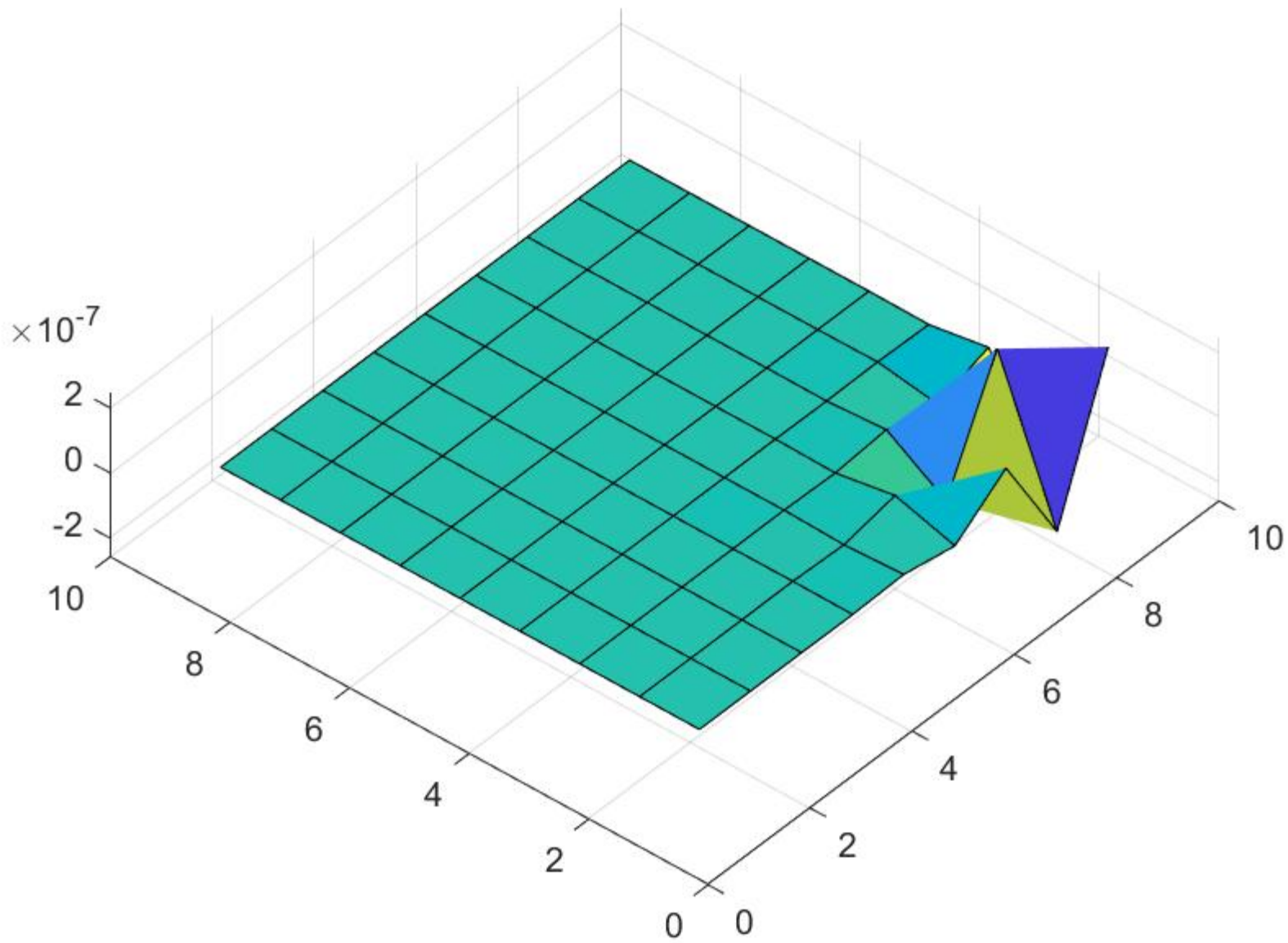
>> eig(F)

0. 044802721845067  
0. 082015890201792  
0. 162582729486328  
0. 374121140912600  
0. 9999999999999966  
2. 672931012564413  
6. 150714797051293  
12. 192759202150013  
22. 320072505788492

>> eig(B)

0. 061039353359070  
0. 061041134158948  
0. 168054858422710  
0. 373370423277969  
1. 000016557229929  
2. 672931171283966  
6. 150714794349185  
12. 192759202129578  
22. 320072505788680







```
>> norm(B-F)
```

```
ans =
```

```
5.0121e-07
```

Wilkinson(9)

```
>> W = wilkinson(9)
```

```
W =
```

```
    4    1    0    0    0    0    0    0    0    0
    1    3    1    0    0    0    0    0    0    0
    0    1    2    1    0    0    0    0    0    0
    0    0    1    1    1    0    0    0    0    0
    0    0    0    1    0    1    0    0    0    0
    0    0    0    0    1    1    1    1    0    0
    0    0    0    0    0    1    2    1    1    0
    0    0    0    0    0    0    1    3    1    0
    0    0    0    0    0    0    0    1    4    0
```

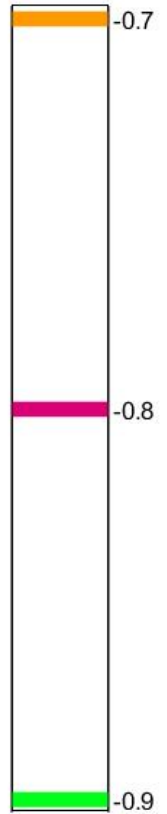
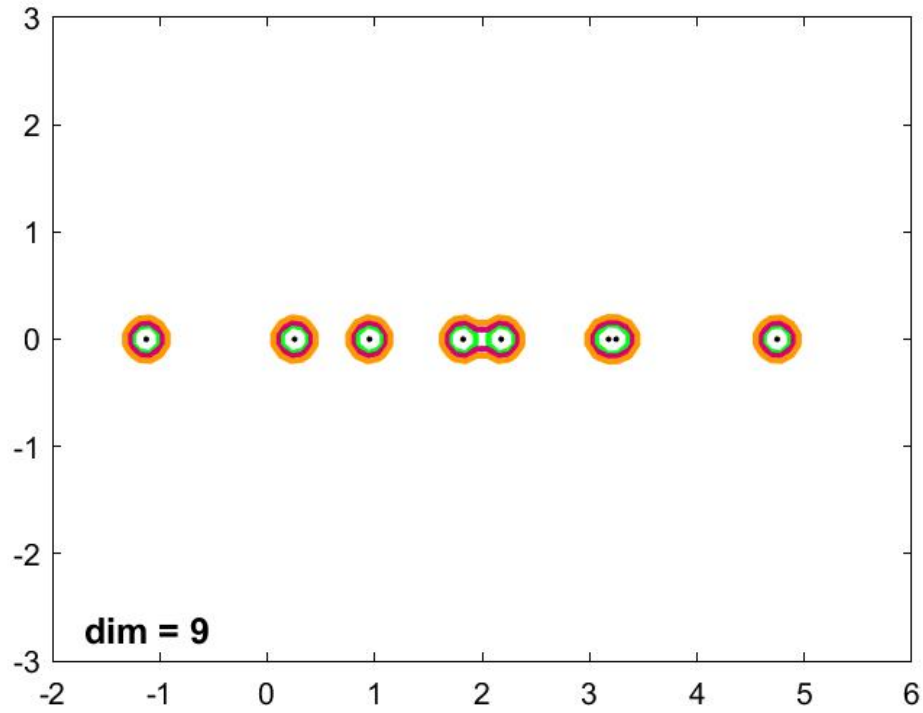
```
>> eig(W)
```

```
- 1.125422415673319  
 0.254718759825861  
 0.952584219075215  
 1.822717080887109  
 2.178284739549981  
 3.177282919112892  
 3.247396472578982  
 4.745281240174140  
 4.747156984469142
```

```
>> condeig(W)
```

```
1.0000000000000000  
1.0000000000000000  
1.0000000000000000  
1.0000000000000000  
1.0000000000000000  
1.0000000000000000  
1.0000000000000000  
1.0000000000000000  
1.0000000000000000
```

# EigTool: left button to zoom in, right button to zoom out



Field of Values
   

  
 Mesh:
   
 Grid Size:

Figure Axes:

Scale Equal

Y max:

X min:  X max:

Y min:

Direct/Iterative:

Direct

ARPACK/eigs

No. eigs (k):

Large Mod.

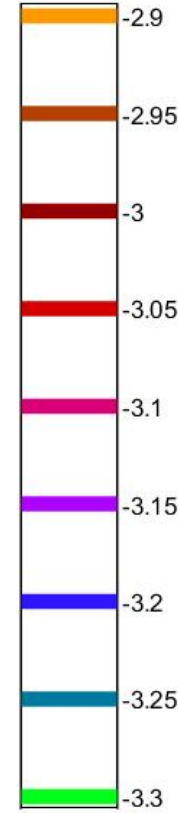
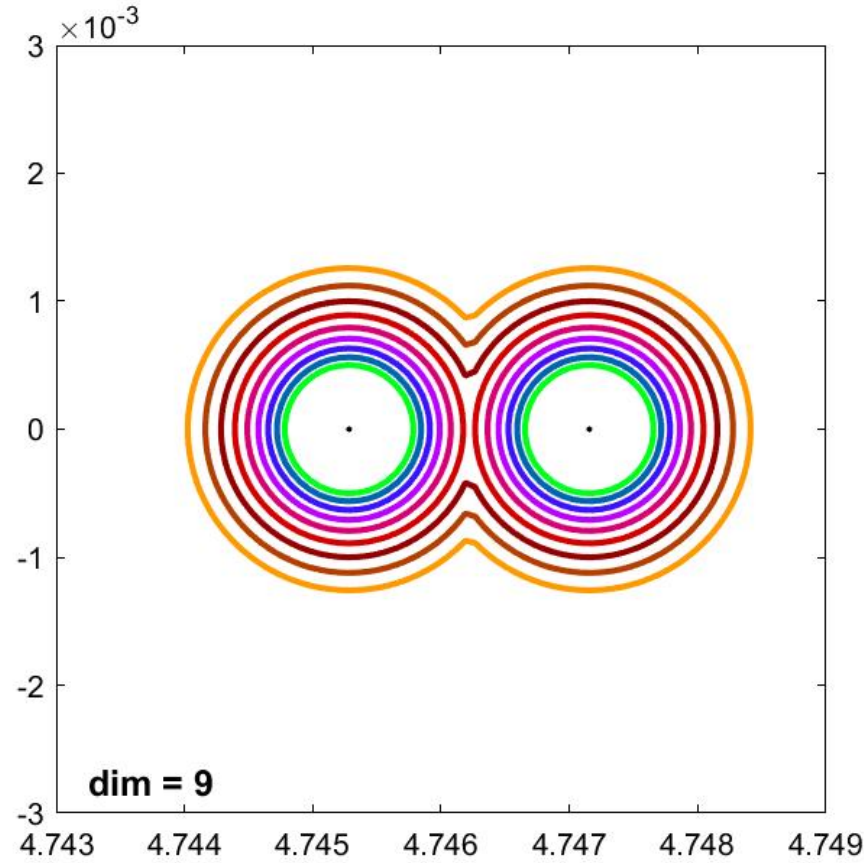
Contour Levels:

log10(largest):

log10(smallest):

Step size:

# EigTool: left button to zoom in, right button to zoom out



Field of Values
   

  
 Mesh:
   
 Grid Size:

Figure Axes:

Scale Equal

Y max:

X min:  X max:

Y min:

Direct/Iterative:

Direct

ARPACK/eigs

No. eigs (k):

Large Mod.

Contour Levels:

log10(largest):

log10(smallest):

Step size:

```
>> B = neardefmat(W);
```

```
neardefmat: A is normal within tolerance
```

```
distance to nearest defective matrix B is 0.000937872
```

```
neardefmat: two closest computed eigenvalues of B differ by 2.01677e-09  
and condition numbers of these eigenvalues are 465036, 465036
```

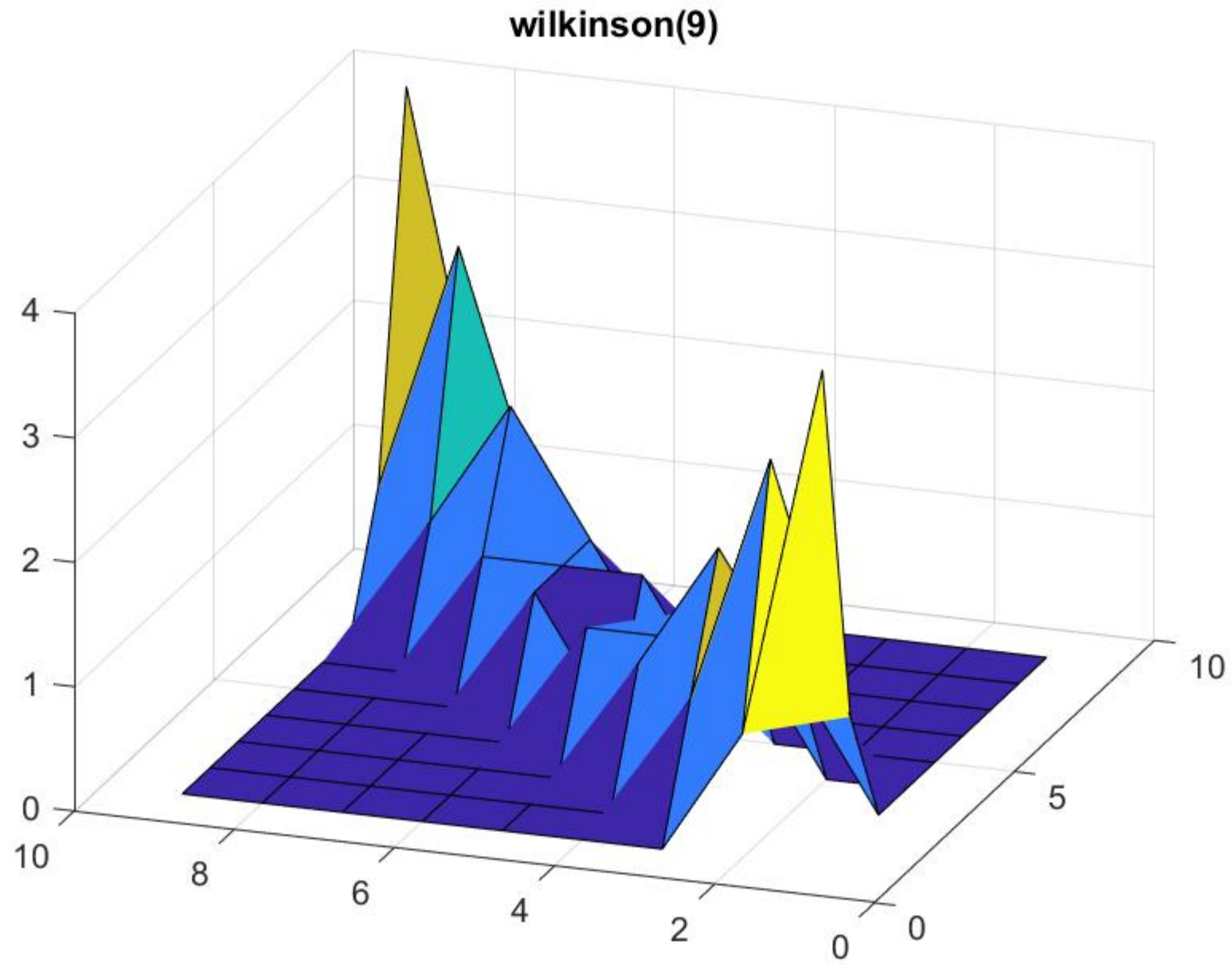
>> eig(W)

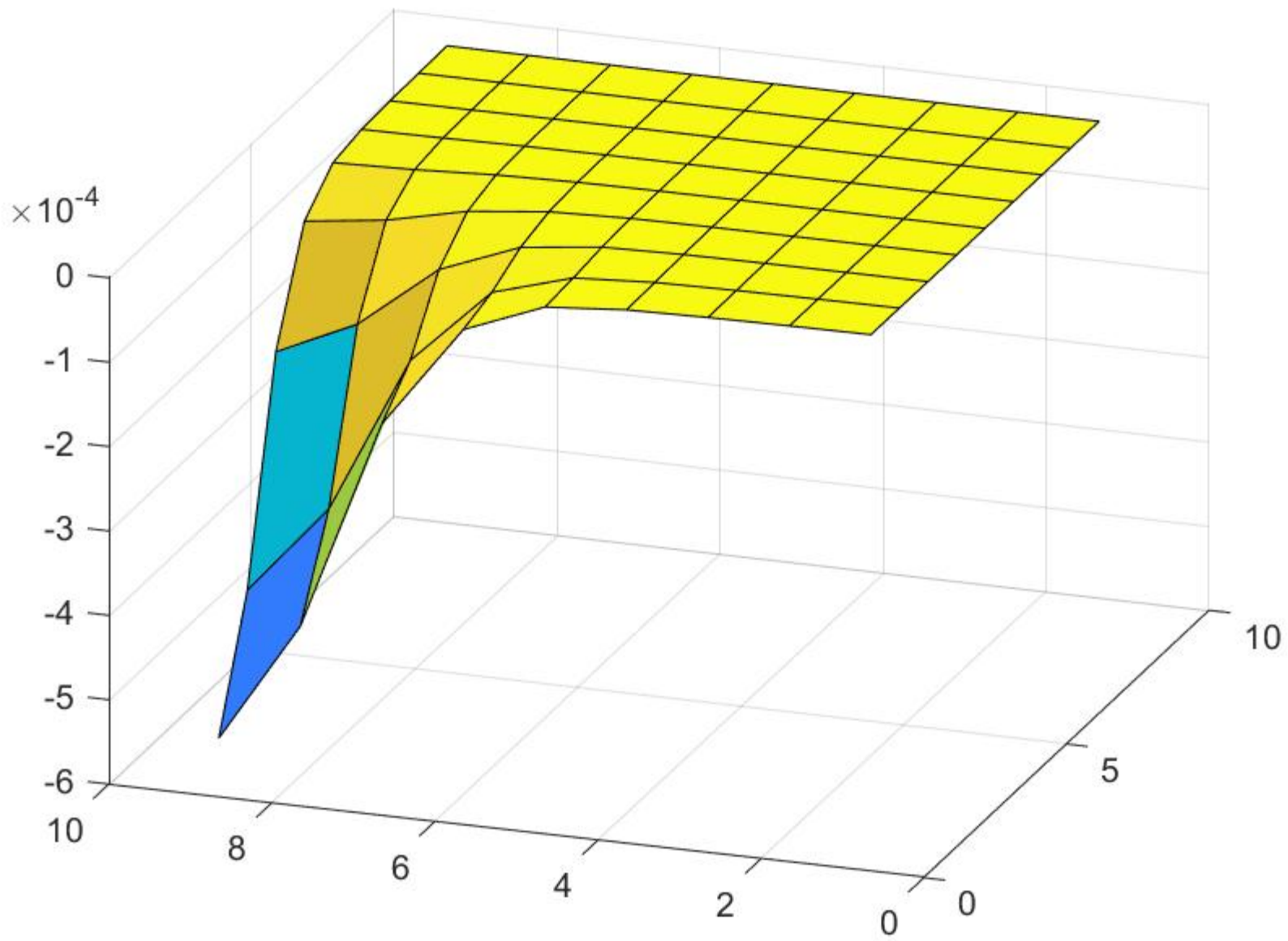
```
- 1.125422415673319
  0.254718759825861
  0.952584219075215
  1.822717080887109
  2.178284739549981
  3.177282919112892
  3.247396472578982
  4.745281240174140
  4.747156984469142
```

>> eig(B)

```
- 1.125422415673318 + 0.000000000000000000i
  0.254718759825861 + 0.000000000000000000i
  0.952584219075214 + 0.000000000000000000i
  1.822717080887109 + 0.000000000000000000i
  2.178284739549983 + 0.000000000000000000i
  3.177282919112890 + 0.000000000000000000i
  3.247396472578982 + 0.000000000000000000i
  4.746219112321641 + 0.000000001008386i
  4.746219112321641 - 0.000000001008386i
```







```
>> norm(B-W)
```

```
ans =
```

```
9.3787e-04
```