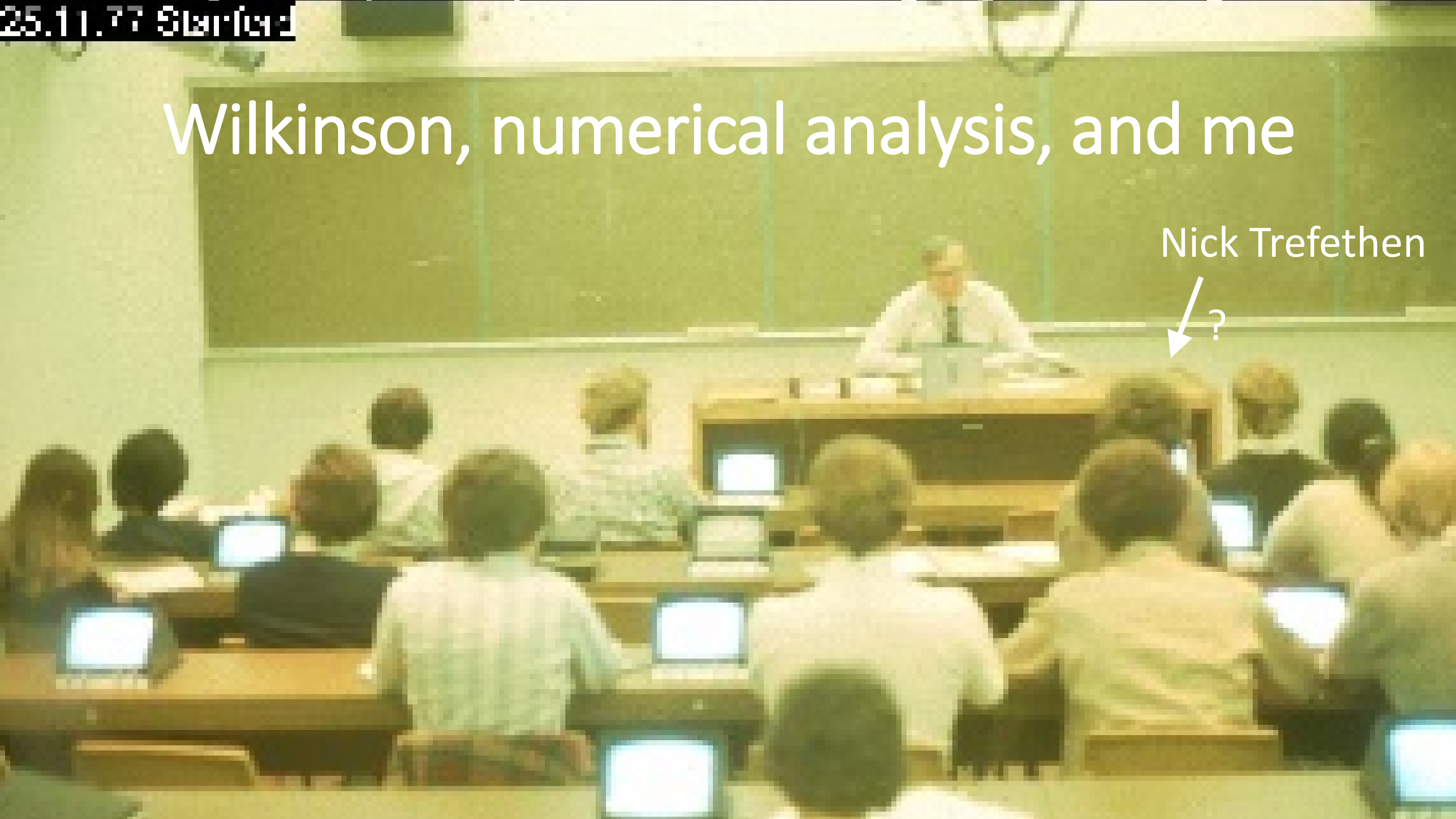
Wilkinson, numerical analysis, and me



Diary notes from Stanford, Fall 1977.
Assessments of 3 Turing Award winners.

Friday 14 October	Lecture by Kahan on floating point implementation. Serra House lunch with Kahan -- he is quite some character.
Friday 28 October	CS237. Wilkinson is wonderful.
Tuesday 22 November	CSD colloquium: McCarthy. Terrible.

13 November 1977

At last Thursday's lunch Wilkinson, Householder, and Golub were all very present, and they are the Hegel, Kant, and Wittgenstein of numerical linear algebra.

If I hadn't loved and respected Serra House already, I must certainly have begun to do so at the colloquium on Thursday. A young man from Belgium was speaking about his pet method for the resolution of singular pencils, and at one point he introduced a certain form of singular value decomposition. "Is it all right," he hesitated, "to go ahead and use singular value decompositions? You are familiar with this?"

"Oh," Gene Golub assured him, "this is the World Headquarters for the Singular Value Decomposition."

"Yes, there will be a small fee for using it," said Wilkinson.

Wilkinson	Numerical analysis
1. Backward error analysis	*Is interval arithmetic useful?*
2. Rounding errors	*What is numerical analysis?*
3. Gaussian elimination	*Is probability important for NA?*
4. Roots of polynomials	*What is mathematics?*

1. Backward error analysis *Is interval arithmetic useful?*

Wilkinson's great discovery was that certain errors are "diabolically correlated".
You can compute $A = LU$ and find both L and U are terrible, yet LU is accurate. Astonishing!
In other words, the computed LU factorization is *the exact solution to a slightly perturbed problem*.

For years this led me to assume interval arithmetic must be useless.
It would tell you, correctly, that L and U are awful.
How could it possibly detect that your solution to $Ax = b$ was nevertheless accurate?

I was mistaken. Interval arithmetic can be used a posteriori to validate x.
You don't track the accuracy of the computation as it proceeds; you check the accuracy of the result.
"Validated numerics". "Reliable computing".
See e.g. Rump, *Acta Numerica* 2010 and INTLAB software and Tucker, *Validated Numerics*.

2. Rounding errors *What is numerical analysis?*

Wilkinson told the world that rounding errors may make computed solutions worthless.

He was the world's best-known numerical analyst, and the world got the message loud and clear. (Forsythe and others were important too.)

Sadly, I believe this has a lot to do with the dim reputation of numerical analysis ever since. People regard NA as an ugly business of engineering compromises. Many believe that *the most important thing to know about NA is that things often go wrong.*

70 Turing Awards winners so far....

In fact, NA is the study of algorithms for problems of continuous mathematics.
Many of the problems are in an exact sense unsolvable, yet we solve them with amazing speed.
This was the subject of my essay "The definition of NA" and of my inaugural lecture at Oxford.

Two stories. David Gries, planning a lecture for the intro-to-CS course, Cornell 1992.
Alain Goriely, writing *A Very Short Introduction to Applied Mathematics*, Oxford 2017.

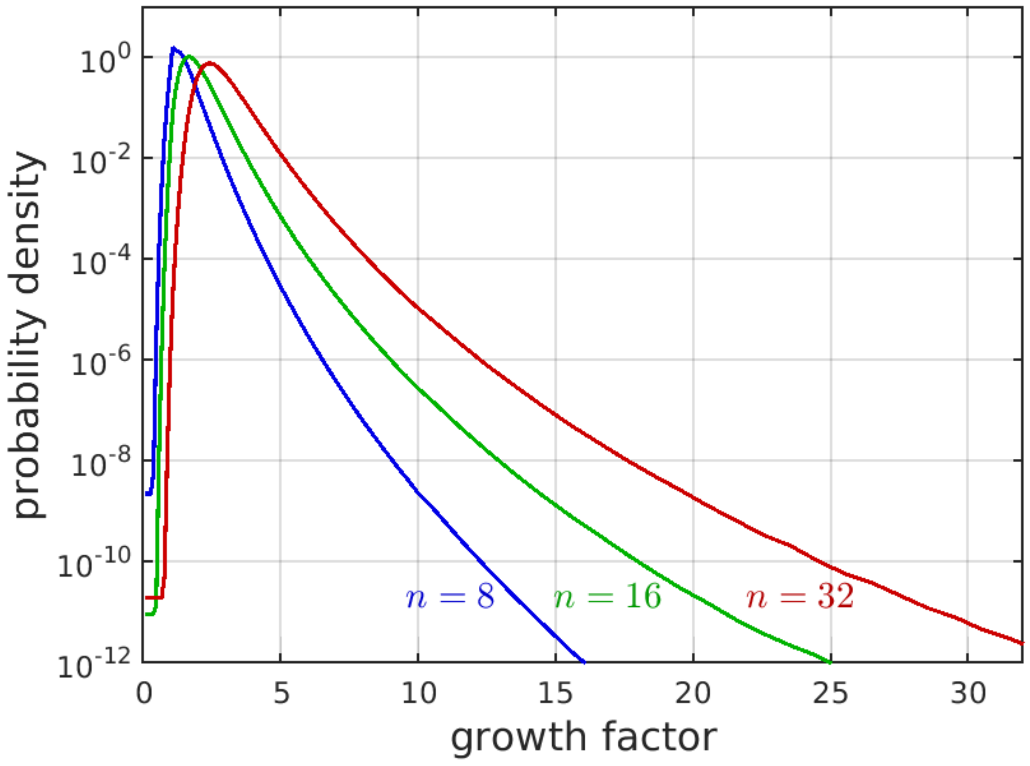
3. Gaussian elimination *Is probability important for NA?*

One of W.'s triumphs was his 1961 analysis of Gaussian elimination. Von Neumann and Hotelling had predicted exponential growth of errors. W. showed this can't occur unless you get exponentially large growth factors.

But you *can* get exponentially large growth factors! Yet somehow in practice they never seem to turn up. ("Anyone that unlucky has already been hit by a bus.")

The explanation is probabilistic. With random matrices, *large growth factors are exponentially rare* (not just algebraically). Unproved, but clear from experiments.

Traditionally, W. and other numerical analysts have little interest in probability, and few people except me have regarded stability of GE as a big unsolved problem.
Is probability moving mainstream in the era of big data?



Driscoll and Maki, *SIREV*, 2007

4. Roots of polynomials

What is mathematics?

One of Wilkinson's core topics was computing roots of polynomials.
I think a key event in this subject happened 130 years before his time.

For centuries, polynomial roots were the central problem of mathematics.
The ancient Greeks solved $n = 2$, the 16th century Italians $n = 3$ and $n = 4$.
But Ruffini (1799) and Abel (1824) proved there is no formula for $n \geq 5$.

Then Galois (1830) opened up a new universe of modern algebra.
I think of this as the moment at which pure mathematics diverged from applied.

> *" It was a total revolution: algebra ceased to have as objective the solution of equations and instead turned to the characterization of different structures. In so doing, this constituted the passage to modern mathematics. "* - Corbelan, *Galois*, 2000

Did rootfinding cease to be important in 1830? Of course not.
But it ceased to be of interest to most mathematicians.
By the time Jim came along, the problem had belonged to the numerical analysts for generations.
What does this say about mathematics?

The 20th c brought us Galois on steroids: algebraic geometry = roots of multivariate polynomials.
Structures, of course, not computations. (In fact, multivariate polynomials are little used numerically.)

Back to Jim.

Wilkinson to give Forsythe lecture

A distinguished British mathematician often called "the world's greatest numerical analyst," Dr. James H. Wilkinson, will present the first of the newly established [...] tributed fundamental research in numerical linear algebra, and his paper on error analysis of linear algebraic equations is considered a classic [...]

Sunday, 13 November 1977

<u>Wilkinson's Forsythe lectures</u>. This past week Wilkinson gave the first annual Forsythe lectures, commemorating George Forsythe with some funds donated by his widow, who is a presence around Serra House and who in fact is having Prof. and Mrs. Wilkinson live with her for the quarter. These first two of three lectures were about the Pilot Ace computer, a somewhat unusual machine designed by Turing and Wilkinson and others right after the war, finally built in 1950.

Wilkinson was sublime. This was one of those talks in which one is spellbound by the sense that a bit of history is being unfolded. Wilkinson pretty much singlehandedly invented numerical linear algebra on that machine in the few years after 1950. He described in his marvelous and humorous style his maverick three-day solution of a system of linear equations in eight unknowns on a hand machine by Gaussian elimination — which lead to an absolutely unexpectly small residual and provided him with the first gleam of insight into backwards error analysis. He described...

These were two grat lectures. Wilkinson is a delight to everyone, a man who helps make academic life worth living.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